TEMPERATURE FIELD IN TWO-LAYER THERMAL SHIELDING ACTING AGAINST A PULSED FLOW OF HEAT

A. M. Fain and A. Yu. Nul'man

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In order to find the temperature field in a two-layer shield designed to resist pulsed heat flows, a modification of the method of weighted residues is employed, together with the heat-balance equation. Linear and nonlinear formulations are considered.

The operation of a large number of modern systems working under conditions of brief, pulsed thermal actions depends on the possibility of ensuring reliable heat shielding for devices extremely sensitive to temperature changes. Two opposing demands are usually laid upon the shielding: a minimum weight and size, on the one hand, and a high stability of temperature conditions behind the barrier, on the other. An effective solution frequently employed is a two-layer thermal shield (Fig. 1) with a thermal resistance at the boundary D between the layers; the outer layer A is made of thin metal sheet and the inner layer B, of heat-resistant plastic. The screening layer A, which may be regarded as a plate with an extremely small Biot number, provides external heat release. The thickness and material of the main layer B are chosen in such a way as to ensure that the temperature of the wall E should remain practically unchanged under the action of an external pulsed thermal flux.

High working temperatures producing thermal radiation, substantial temperature gradients, and requiring allowance to be made for variations in the physical properties of the materials, together with the foregoing requirements imposed upon the heat shield, in general exclude the possibility of linearization. However, even in the linear formulation the solution obtained by classical methods becomes unstable owing to the necessity of allowing for many terms of the expansion, which is inevitable in the case of pulsed thermal action.

In this paper we shall consider a method of approximately determining the temperature field in a two-layer shielding such as will give a high engineering accuracy, stability of the solution, and practically identical amounts of computing work for the linear and nonlinear approaches.

Linear Problem. Let a heat flow $q(\tau)$ fall on the surface C, reaching a maximum q_{max} at the instant of time $\tau = \tau_{max}$. The original heat-conduction equation and boundary conditions are specified in the form

$$\dot{T} - aT''_{xx} = 0; \quad (1)$$

$$\bar{T} - T + R\lambda T'_{xx}|_{x=0,\ \tau} = 0; \quad (2)$$

$$T'_x|_{x=\delta,\ \tau=0} = \bar{T}|_{\tau=0} = T|_{x,\ \tau=0} = 0; \quad (3)$$

$$q - \bar{c}\bar{\gamma}\bar{\delta}\dot{\bar{T}} - \alpha T + \lambda T'_x|_{x=0,\ \tau} = 0. \quad (4)$$

Equation (2) expresses the temperature jump at the boundary between the layers; condition (4) represents the heat-balance equation. Let us introduce the following dimensionless coordinates and notation:

$$\begin{gathered} \psi = x/\delta;\ t = T/T_0;\ \eta = \tau/\tau_{max}; \\ \mathrm{Fo} = a\tau_{max}/\delta^2;\ \bar{t} = \bar{T}/T_0;\ f = q/q_{max}; \\ k_\eta = \bar{c}\bar{\gamma}\bar{\delta}T_0/(q_{max}\tau_{max});\ k = \alpha T_0/q_{max}; \\ k_\psi = \lambda T_0/q_{max};\ k_{\eta\psi} = k_\eta R/\delta;\ \bar{k}_\psi = R\lambda/\delta. \end{gathered} \quad (5)$$

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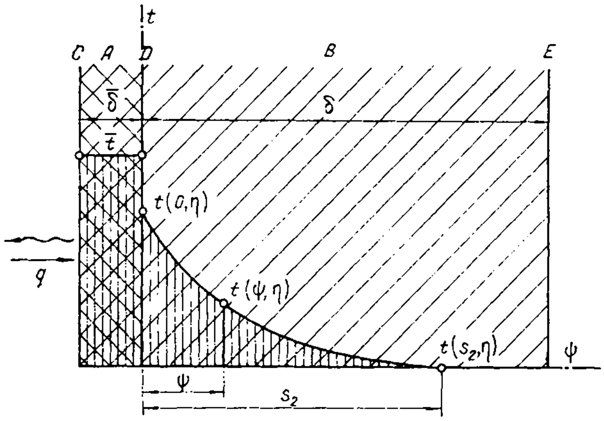

Fig. 1

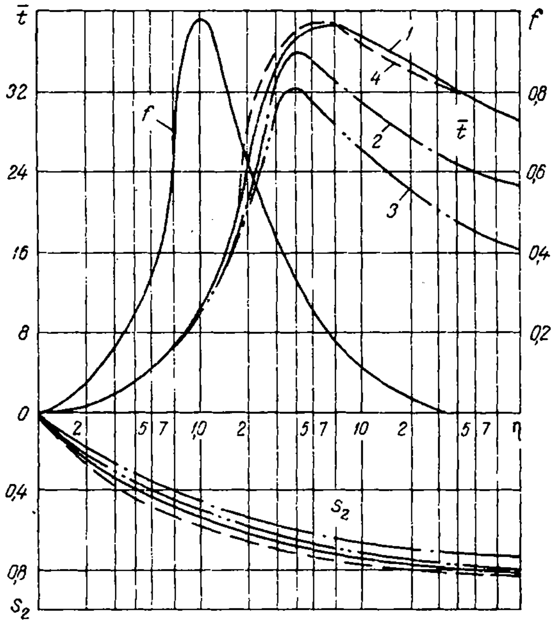

Fig. 2

Fig. 1. Computing scheme for two-layer shielding.

Fig. 2. Example of calculation: f, function of thermal flux, dimensionless; $\bar{t}$, temperature of layer A, dimensionless; s_2, effective depth of heating, dimensionless; 1) linear problem (boundary condition of the third kind); 2) nonlinear problem (radiation by the Stefan—Boltzmann law); 3) nonlinear problem (radiation by the Stefan—Boltzmann law, variable thermal conductivity); 4) linear problem (comparative solution by the method of iterations).

In the notation of (5) the original problem (1)-(4) takes the form

$$L(t) \equiv \dot{t} - \mathrm{Fo}\, t''_{\psi\psi} = 0; \tag{6}$$

$$M(t,\ \psi = 0) \equiv f - k_\eta \dot{\bar{t}} + k_{\eta\psi} \dot{t}'_\psi - kt + k_\psi t'_\psi|_{\psi=0,\eta} = 0; \tag{7}$$

$$D(t,\ \psi = 0) \equiv \bar{t} - t + \bar{k}_\psi t'_\psi|_{\psi=0,\ \eta} = 0; \tag{8}$$

$$\bar{t}|_{\eta=0} = t|_{\psi,\ \eta=0} = 0. \tag{9}$$

According to definition, the thermal flux has two monotonic regions (Fig. 2): rising for $\eta < 1$ and falling for $\eta > 1$. Naturally, on allowing for heat transfer to the ambient the temperature field should have two monotonic regions; the maximum temperature of layer A will lag with respect to the maximum thermal flux, usually by a time much greater than the duration of the external pulse. It is precisely this delay which leads to the extremely difficult and not always soluble problem of overcoming the instability of computer calculations in obtaining practical results. This difficulty may be avoided by considering the original problem for the two time intervals I ($0 \le \eta \le 1$) and II ($\eta > 1$). Here $t_I = t_1$, $f_1 = f$ and $t_{II} = t_2 - t_3$, $f_2 = 1$, $f_3 = 1 - f$. In this case, first, t_0, f_j ($j = 1, 2, 3$) will be monotonic functions, and the fundamental possibility of instabilities appearing in the solution will vanish; secondly, t_j should satisfy (6)-(8) and the initial conditions

$$t_1|_{\psi,\ \eta=0} = \bar{t}_1|_{\eta=0} = t_3|_{\psi,\ \eta=1} = \bar{t}_3|_{\eta=1} = 0;$$
$$t_2|_{\psi,\ \eta=1} = t_1|_{\psi,\ \eta=1};\ \bar{t}_2|_{\eta=1} = \bar{t}_1|_{\eta=1}. \tag{10}$$

In estimating the merits of different versions of the shielding we must know three defining parameters: the temperature of layer A — $\bar{t}(\eta)$; the maximum temperature of layer B — $t(0,\ \eta)$; and the time during which the temperature of wall E is no greater than a set limit. Taking account of (8) it is sufficient to determine simply the first and third parameters. Taking these as minimizing functions $s_1(\eta)$, $s_2(\eta)$ and using the well-known idea of a finite heating depth [1], we shall seek the approximate solutions $\tilde{t}_j$ in the form

$$\tilde{t}_j = s_{1j}(1 - \psi/s_{2j})^2. \tag{11}$$

We replace the differential equation (6) by the integrated equation [2]

$$\int_0^{s_{2j}} L(\tilde{t}_j)\, \frac{\partial \tilde{t}_j}{\partial s_{1j}} = 0. \tag{12}$$

Allowing for (11), conditions (7) and (12) may be transformed and reduced to the normal form:

$$s_{ij} = \begin{vmatrix} \varphi_{11,j}\varphi_{12,j}\varphi_{10,j} \\ \varphi_{21,j}\varphi_{22,j}\varphi_{20,j} \end{vmatrix}; \qquad s_{i0}(0) = s_{i3}(1);\ s_{11}(1) = s_{12}(1);\ s_{22}(1) = s_{21}(1), \tag{13}$$

where

$$\varphi_{11.j} = k_\eta + k_{\eta\psi} + 2/s_{2j};\ \varphi_{12.j} = -2k_{\eta\psi}s_{1j}/s_{2j}^2; \quad \varphi_{10.j} = f_j - s_{1j}(k + 2\,k_\psi/s_{2j}); \quad \varphi_{21.j} = 2s_{2j}^2;\ \varphi_{22.j} = s_{1j}s_{2j};\ \varphi_{20.j} = 6.67\mathrm{Fo}\, s_{1j}. \tag{14}$$

Solution (13) enables us to find the unknowns s_{ij} and hence the unknown temperature field in practically any computer.

Nonlinear Formulation

On allowing for thermal radiation from the surface C obeying the fourth-power law the thermal balance condition takes the form

$$q - \bar{c}\bar{\gamma}\delta\dot{\bar{T}} - \alpha T - \sigma\bar{T}^4 + \lambda T'_x|_{x=0,\,\tau} = 0. \tag{15}$$

The heat-conduction equation (6) remains as before, and so do conditions (8) and (9). Let us denote

$$k_\sigma = \sigma T_0^4/q_{\max}; \quad F(t,\ \psi = 0) = -k_\sigma(t - k_\psi t'_\psi)^4|_{\psi=0,\ \eta=0} = 0. \tag{16}$$

Condition (15) expressed in the notation of (7) and (16) then takes the form

$$M(t,\ \psi = 0) - F(t,\ \psi = 0) = 0. \tag{17}$$

Seeking the solution in the same way as in the linear problem, for the same monotonic regions I and II we find that t_1 and t_2 should satisfy (6) and (17), while t_3 should satisfy Eq. (6) and the condition

$$M(t_3,\ \psi = 0) - F(t_3,\ \psi = 0) - F(t_2 - t_3,\ \psi = 0) = 0, \tag{18}$$

of which it is quite easy to convince oneself by subtracting (18) from (17) and considering that $t_{II} = t_2 - t_3$. The boundary conditions retain the form (10).

Taking the form (11) for $\tilde{t}_j$ as before, from conditions (12), (17), and (18) we arrive at the system (13), in which only one free term differs from (14) and is equal to

$$\varphi_{10,j} = f_j - s_{1j}(k + 2k_\psi/s_{2j}) - G_j, \tag{19}$$

in which

$$G_j = k_\sigma s_{1j}^4 (1 + 2\bar{k}_\psi/s_{2j})^4 \ \text{ for } \ j = 1,\ 2; \quad G_3 = G_2 - k_\sigma s_{1j}^4 [s_{12}/s_{13} - 1 + 2\bar{k}_\psi(s_{32} - s_{22})/(s_{22} - s_{32})]^4. \tag{20}$$

In order to be specific, let us consider the boundary condition with the thermal radiation obeying the fourth-power law, the thermal conductivity being a linear function of temperature $\lambda = \lambda_0(1 + \beta T)$.

In the notation adopted, the original problem may be written as follows:

$$P(t) \equiv \dot{t} - \mathrm{Fo}\,[(1 + \beta_T t)t'_\psi]'_\psi = 0 \quad (\beta_T = \beta T_0); \tag{21}$$

$$f - \dot{t}(k_\eta - k_{\eta\psi}\beta_T t'_\psi) - k_{\eta\psi}\dot{t}'_\psi(1 + \beta_T t) - k_\psi t'_\psi(1 + \beta_T t) + k[t - k_\psi t'_\psi(1 + \beta_T t)]^4|_{\psi=0,\ \eta} = 0 \tag{22}$$

subject to boundary conditions (10). As in the two earlier problems we consider two time intervals, replacing (21) by the integrated condition

$$\int_0^{s_{2j}} P(\tilde{t}_j) \frac{\partial \tilde{t}_i}{\partial s_{1j}} d\psi = 0; \tag{23}$$

substituting t_j in (22) and making some simple transformations, we again arrive at Eq. (13), in which $\varphi_{ik,j}$ are simply distinguished by having a more complicated form.

Allowance for thermal radiation and a variable thermal conductivity (other conditions being equal) leads to the appearance of strongly expressed maxima considerably displaced to the left hand side. Like the laws governing the heating of B, this result is in excellent agreement with physical considerations.

A comparative calculation carried out for the linear problem by the considerably more troublesome method of iterations revealed excellent agreement between the results (Fig. 2).

NOTATION

$\bar{T}$, T, temperatures of layers A and B; $\bar{c}$, c, specific heats; $\bar{\delta}$, δ, thicknesses; x, coordinate; q, thermal flux; τ, time; a, thermal diffusivity; R, thermal resistance; σ, reduced emissivity; ψ, dimensionless coordinate; η, dimensionless time; f, dimensionless thermal flux; $\bar{t}$, t, dimensionless temperatures; Fo, Fourier number; s, minimizing function.

LITERATURE CITED

1. T. Goodman, in: Heat-Transfer Problems [Russian translation], Atomizdat, Moscow (1967), p. 41.
2. L. V. Kantorovich and V. I. Krylov, Approximate Methods of Higher Analysis [in Russian], FM, Moscow (1962).